

A Note on How NK Landscapes Work

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Abstract

The NK landscape methodology has been used by much research in strategy and organizations, and the concept of “landscape” has become a popular business idea. Despite such popularity, exactly what NK landscapes are and how they work is typically obscure to all but a small specialist audience. This technical note clarifies the NK landscape methodology by explaining how an NK landscape is computed. This note also discusses ways in which NK landscapes are represented and used in research. The aim of this note is enabling more researchers to engage more deeply with the work that relies on the landscape concept.

INTRODUCTION

NK landscapes are commonly mentioned in the strategy and organizations literatures. In fact, over 70 published management papers have developed NK models (see the excellent survey by Baumann, Schmidt, and Stieglitz 2018) and many articles and books use the idea of “landscape” to talk about phenomena such as search, exploration, imitation, competition, fit, and organization design (the “landscape” idea has even made it to the cover of an MBA strategy textbook; see Ghemawat 2010).

Despite this popularity, there is much misunderstanding regarding NK landscapes and how they operate. For example, many believe that NK landscapes are simply a three-dimensional surface (like the ones in figures 2a and 2b). This state of affairs is probably due to the page-length constraints of journals—a 30-page paper cannot devote five pages to describing its base model. Hence, every author of an NK paper hopes that its readers will already understand NK landscapes at the necessary level, while in reality very few do. Typically the only who can engage at a deep level with an NK paper are those that have developed their own NK models. This is an unfortunate scenario; it is like the only who could understand a linear regression were those who have programmed their own. It is also an unnecessary state of affairs, as NK landscapes are simple mathematical objects, which only require basic arithmetic to be understood.

The goal of this short note is to explain clearly how an NK landscape is computed. Understanding this is useful, as a similar computation is shared by all NK models. Most NK models do not differ significantly in how the landscape is constructed but on the search process that occurs on the landscape (hence, papers usually only explain in detail their unique search process). However, all the results in an NK paper stem from the interaction between the

landscape and the search process (in this way, NK models are an embodiment of “Simon’s scissors” principle; Simon 1990). Thus, having a clear understanding of how the landscape is constructed is necessary to understand the results of an NK paper. The goal of this note is to serve as a reference of this core component of all NK papers. The hope of this note is to enable more researchers to engage at a deeper level with the work that relies on the landscape concept.

UNDERSTANDING THE NK LANDSCAPE METHODOLOGY

The NK landscape methodology allows modeling the performance of a general class of systems. Although originally developed to model biological systems (Kauffman and Levin 1987; Kauffman 1993), the NK methodology has been used extensively to model firms (e.g., Levinthal 1997) and products (e.g., Sommer and Loch 2004). Arguably, this methodology has been appealing to modelers of organizations, as it provides novel ways to formally analyze core organizational issues such as bounded rationality, modularity, interdependence, and organizational search.

The NK landscape methodology encompasses a family of models. This note explains how to compute a standard NK landscape and gives pointers to papers that have extended the standard model.

An NK landscape is a function that maps the state of a system onto a measure of its performance, which is customarily called *fitness*. The system is assumed to have N components, and each component can exist in a number of states. For example, imagine a portable computer made of $N = 3$ components—screen, battery, and CPU—and suppose that each component can exist in one of two states: the screen can be small or large, the battery can be low-capacity or high-capacity, and the CPU can be slow or fast.

The contribution of a given component to the fitness of the system depends not only on the state of that component but also on the states of the other components with which it interacts. In the case of our computer: the screen's contribution to fitness depends on its size and on CPU speed (e.g., the combination of a large screen and a slow CPU yields especially sluggish performance); the battery's contribution to fitness depends on its capacity and on screen size (e.g., batteries are more rapidly depleted by larger screens); and the CPU's contribution to fitness depends on its speed and on the battery's capacity (e.g., a fast CPU works poorly with a weak battery).

The components and interactions in an NK model can be equivalently represented as a directed graph or an interaction matrix. See Figure 1 for the directed graph and interaction matrix corresponding to the computer example mentioned above.

INSERT FIGURE 1 ABOUT HERE

In the *directed graph* representation, each box denotes a component and each arrow an interaction. In the *interaction matrix* representation, cell (i, j) contains a 1 if component i depends on component j (i.e., if the graph representation includes an arrow from j to i). Because the fitness of a component depends on its own state, the diagonal of the matrix is filled with ones.

How much each component contributes to the product's overall fitness is described by the N contribution functions (i.e., one for each component). The *contribution function* c_i maps the states of the components that affect the contribution of component i onto the fitness contribution of component i . Continuing with our computer example, since the screen's

contribution to fitness depends on screen size and CPU speed, and since each component can be in two different states, it follows that the screen’s contribution function can take one of four values: $c_{\text{screen}}(\text{small, slow})$, $c_{\text{screen}}(\text{small, fast})$, $c_{\text{screen}}(\text{large, slow})$, and $c_{\text{screen}}(\text{large, fast})$.

The standard NK model assumes that each component can be in either of two states (0 and 1) and that each component depends on K other components (i.e., that the interaction matrix has $K + 1$ ones per row). Thus, the ongoing example has $K = 1$ as each component depends on one other component apart from itself. Parameter K controls the degree of interdependence or interaction among the system’s different components. When $K = 0$, each component is independent of the others; when K is maximal ($K = N - 1$), each component depends on every other component. With higher K , it is more likely that changing the state of one component will have an effect on the contribution of other components. This parameter can be seen as a way to “tune” the *complexity* that underlies the mapping between choices and fitness.

The fitness f of a given position is defined as the sum of the fitness contributions for that position (normalized by N so that fitness values will be comparable across landscapes of different N ’s). Namely,

$$f(\mathbf{s}) = \frac{1}{N} \sum_{i=1}^N c_i(s_i; K \text{ other elements of } \mathbf{s}),$$

where $\mathbf{s} = (s_1, \dots, s_N)$ represents the state of each component of the system.

To illustrate how fitness is computed, suppose the contribution functions of the three components in our example are as follows:¹

¹ The contribution values used in this example are arbitrary.

Contribution of screen	Contribution of battery	Contribution of CPU
<u>screen</u> <u>CPU</u>	<u>battery</u> <u>screen</u>	<u>CPU</u> <u>battery</u>
$c_1(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}) = 0.9$	$c_2(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}) = 0.8$	$c_3(\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}) = 0.1$
$c_1(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}) = 0.5$	$c_2(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}) = 0.0$	$c_3(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}) = 0.5$
$c_1(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}) = 0.3$	$c_2(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}) = 0.0$	$c_3(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}) = 0.2$
$c_1(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}) = 0.7$	$c_2(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}) = 0.3$	$c_3(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}) = 0.9$

Thus, the fitness of a portable computer with small screen ($s_1 = 0$), weak battery ($s_2 = 0$), and fast CPU ($s_3 = 1$) would be:

$$f(0,0,1) = \frac{1}{3}(c_1(0,1) + c_2(0,0) + c_3(1,0)) = \frac{1}{3}(0.5 + 0.8 + 0.2) = 0.5.$$

It is sometimes useful to represent NK landscapes as hypercubes, where each node represents a position and each link a connection between neighboring states (states that differ by one element). The hypercube representation of the landscape for our example appears in Figure 2. One can think of hypercubes with more than $N = 3$ dimensions as graphs where each node has N neighbors (see, e.g., Harary, Hayes, and Wu 1988).

INSERT FIGURE 2 ABOUT HERE

Figure 2 can be used to illustrate how a boundedly-rational actor may search in the landscape. Such an actor is usually conceptualized as not being able to “see” the whole landscape. Instead, the actor can only see the positions that are close to its current position and pick the best among these, a process akin to “hill climbing.” In other words, when trying to pick what is the configuration with the highest fitness, a boundedly rational actor cannot just jump to the best position or global peak (in Figure 2 this would be position (1,1,1), which has a fitness of 0.63) but instead needs to search the landscape, at each step deciding to which neighboring position to move. Such a search process may leave the actor stranded at a local peak—a position

that is higher than all its neighbors but is not the global peak. As landscapes grow in complexity, they become more rugged; that is, they contain more local peaks. Whether the agent will find the global peak depends on how it searches (e.g., how far or accurately it can see) and on luck (e.g., its starting position). For instance, in Figure 2, hill climbing from position (1,0,0) would get stuck in position (0,0,0), a local peak (it has fitness 0.60). Instead, if the agent had started in position (1,0,1), hill climbing would reach the global peak.

Although NK landscapes are N -dimensional objects, it is often useful to think about them as three-dimensional surfaces, where the horizontal dimensions represent policy choices, and the vertical dimension represents fitness. Because as K increases a landscape has more local peaks, the equivalent of K in this three-dimensional imagery is the ruggedness of a surface. Figures 3a and 3b illustrate low- and high- K landscapes.

INSERT FIGURE 3 ABOUT HERE

The main property of NK landscapes is that, the more interdependent a system is (i.e., the higher is K), the less similar are the fitness values of neighboring positions. If $K = 0$ then the fitness landscape represents a smooth surface whose global maximum can be found by hill climbing; as K increases, the landscape becomes more rugged or multi-peaked and thus, hill climbing is unlikely to find the global maximum; and if K is maximal ($K = N - 1$) then the landscape represents an extremely rugged or spiky surface on which there is no correlation between the fitness of neighboring points. Bounded rationality in this context is usually conceptualized as a search process that is not omniscient about the whole landscape. Because

few analytic results are known about NK landscapes (for one of the few exceptions, see Durrett and Limic 2003), these models are typically analyzed via simulation.

SOME APPLICATIONS OF THE NK LANDSCAPE METHODOLOGY

The NK landscape methodology has been popular with the strategy and organizations literatures because it captures fundamental ideas of these literatures. Namely, that system-level outcomes depend on the performance of multiple interacting components and, thus, successfully managing such a system requires paying close attention to managers' cognitive and organizational limitations. Paradoxically, although organizations are complex, the NK landscape methodology offers a simple way of investigating such complexity.

Armed with this methodology, researchers have been able to investigate many phenomena, including the search for dominant designs (Levinthal 1997), imitation and replication of strategies (Rivkin 2000; Csaszar and Siggelkow 2010), competition (Lenox, Rockart, and Lewin 2006; Adner, Csaszar, and Zemsky 2014), the use of analogies and other mental representations by managers (Gavetti, Levinthal, and Rivkin 2005; Csaszar and Levinthal 2016), and the ways in which different actors can "shape" the firms' environment (Levinthal and Warglien 1999; Gavetti, Helfat, and Marengo 2017; Li and Csaszar 2018).

The NK landscape methodology has contributed greatly to the organization design literature. Some questions that have been addressed include how different organizational structures affect firms' ability to explore (Siggelkow and Levinthal 2003; Siggelkow and Rivkin 2006; Levinthal and Workiewicz 2018), to escape competency traps (Siggelkow and Levinthal 2005), and to deal with complex and turbulent environments (Ethiraj and Levinthal 2004; Siggelkow and Rivkin 2005). It has also increased our understanding of the effects of different

interaction patterns (Rivkin and Siggelkow 2007) and incentive structures (Ethiraj and Levinthal 2009). The NK methodology is a valuable methodological addition to the organization design toolbox, as it allows delving into the central question of organization design—how different decision-making structures perform under different environments—while avoiding organization design’s severe empirical constraints.

Since its introduction to the strategy and organizations literatures more than 20 years ago (Levinthal 1997), research based on NK models has produced a steady stream of new insights; it is therefore likely that NK models will continue enriching our research landscape.

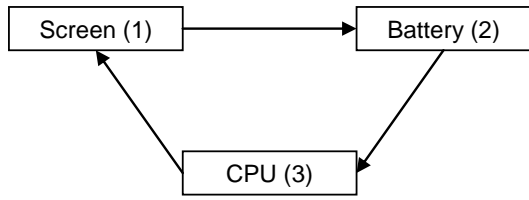
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FIGURES

Figure 1. Directed graph and interaction matrix corresponding to the computer example.



(a) Directed graph

$$\mathbf{I} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \end{matrix}$$

(b) Interaction matrix

Figure 2. Hypercube representation of the NK landscape in the example.

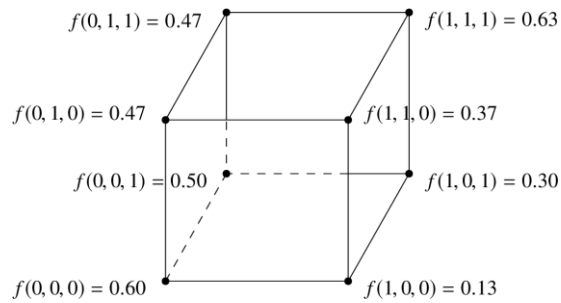
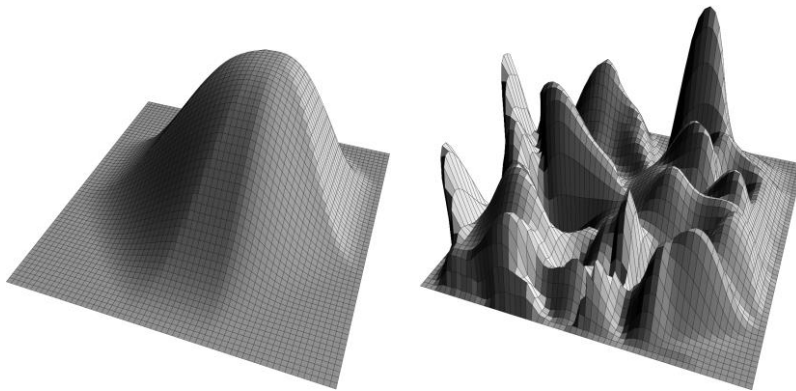


Figure 3. Graphical representations of NK landscapes.



(a) Low K surface

(b) High K surface

