**A Note on Calculating the Average Span of Control**

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January 12, 2021

On pages 168–169, Blau & Scott's (1962) classic book presents a formula for determining the "average span of control of an organization." This formula returns an organization's average span of control (*S*) as a function of the organization's number of employees (*N*) and supervisory levels (*L*) assuming every employee with reporting subalterns has an identical span of control. The average span of control is a useful summary statistic of an organization, as it provides an approximate understanding of the organization's structure and management processes. For instance, if an organization with 1,000 employees had an average *S* of 20, one would know that the organization must be quite flat and that managers would have little time to monitor and interact with their many subalterns. The opposite would be true if one knew that the same organization has an average *S* of, say, 2.3 (for classic articles on span of control, see Ouchi & Dowling 1974 and Van Fleet & Bedeian 1976; for examples of how the concept is used, see Khandwalla 1977, Burton & Obel 2004, and Daft 2010). Referring to the "average span of control" of an organization is common; for example, at the time of writing, Google and Google Scholar return 8,820 and 532 results respectively for the exact term in quotes.

Blau & Scott (1962) provide the formula $S = \sqrt[L]{N}$ without explaining how this formula was derived.[1] It turns out the formula they provide is incorrect. This note provides the correct formula and explains its derivation. Figure 1 will be used to illustrate the derivation. This figure shows an organization with *L* = 2 supervisory levels and a span of control *S* = 2; consequently, this organization has *N* = 7 employees.

More generally, for an organization with span of control *S* and number of supervisory levels *L*, the total number of employees is $N = 1 + S + S^2 + S^3 + \cdots + S^L = \sum_{i=0}^{L} S^i$. That is, *N* is the sum of a geometric progression, which simplifies to:

$$N = \frac{S^{L+1} - 1}{S - 1} \qquad (1)$$

[1] I thank Saerom Lee for pointing this out.

Hence, computing the average span of control $S$ given $N$ and $L$, simply means solving for $S$ in the previous equation. There is no closed-form formula for $S$; hence, to solve for it, one must do so numerically.[2]

The formula that Blau & Scott (1962) provided actually calculates $S$ as a function of $L$ and the number of employees in the lowest layer. For example, in the organization depicted in Figure 1, the number of employees in the lowest layer is 4, which plugged into Blau & Scott's formula gives $S = \sqrt[2]{4} = 2$.

Panels (a) and (b) in Table 1 compare the results of the correct formula vis-à-vis Blau & Scott's for a few selected cases. Panel (c) shows the difference between the two measures as a percentage. One can see that Blau & Scott's formula incurs larger errors when $N$ is small or $L$ is large, as in these cases the organization has proportionally more supervisors and, hence, the number of employees in the lower layer is a bad approximation of the total number of employees.

## References


Blau, P. M., & Scott, W. R. (1962). *Formal Organizations: A Comparative Approach*. San Francisco: Chandler Pub. Co.

Burton, R. M., & Obel, B. (2004). *Strategic Organizational Diagnosis and Design: The Dynamics of Fit*. Boston, MA: Kluwer.

Daft, R. L. (2010). *Organization Theory and Design.* Mason, OH: South-Western.

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Ouchi, W. G. (1977). *Relationship between organizational-structure and organizational control*. Administrative Science Quarterly, 22(1), 95–113.

Van Fleet, D. D., & Bedeian, A. G. (1977). *A history of the span of management*. Academy of Management Review, 2(3), 356–372.


[2] Here is an example of how to compute it using R: `S <- function(N, L) uniroot(function(N,L,S) N-(S^(L+1)-1)/(S-1), lower=1.001, upper=N, N=N, L=L)$root; S(100,4)`

## Figure and Table

**Figure 1:** Illustrative hierarchy.

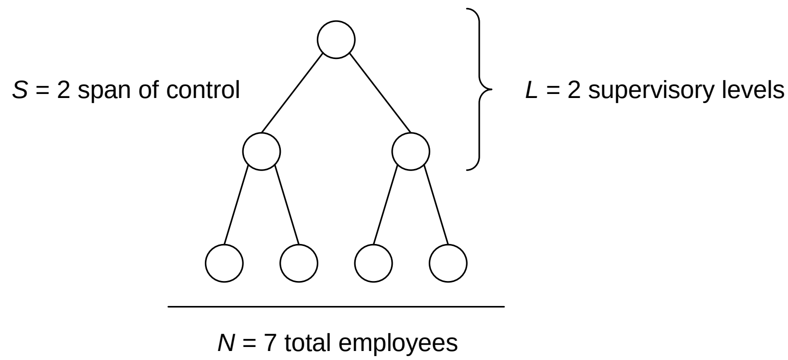


**Table 1:** Panels (a) and (b) compute the corrected measure and Blau & Scott's one for a few selected cases. Panel (c) shows the difference between the two measures as a percentage.

(a) Corrected measure

| | | *L* = 2 | 4 | 6 |
|---|---|---|---|---|
| *N* | 10 | 2.54 | 1.35 | 1.12 |
| | 100 | 9.46 | 2.84 | 1.91 |
| | 1000 | 31.11 | 5.34 | 2.95 |

(b) Blau & Scott's

| | | *L* = 2 | 4 | 6 |
|---|---|---|---|---|
| *N* | 10 | 3.16 | 1.78 | 1.47 |
| | 100 | 10.00 | 3.16 | 2.15 |
| | 1000 | 31.62 | 5.62 | 3.16 |

(c) Error (as %)

| | | *L* = 2 | 4 | 6 |
|---|---|---|---|---|
| *N* | 10 | 24.43 | 31.49 | 31.39 |
| | 100 | 5.68 | 11.30 | 12.98 |
| | 1000 | 1.65 | 5.31 | 7.13 |